Appendix

Proof of Equivalence of Likelihoods

The distribution of $\boldsymbol{Z}^*$ in (3) can be shown to be independent n-variate doubly-truncated standard normal, with density function

$$f_{\boldsymbol{Z}^*}(\boldsymbol{z}) = \gamma^{-1}\exp\left(-\frac{1}{2}\boldsymbol{z}'\boldsymbol{z}\right)$$

where the normalizing constant γ is given by

$$\gamma = \int_{\Phi^{-1}[F_1(y_1-1)]}^{\Phi^{-1}[F_1(y_1)]} \cdots \int_{\Phi^{-1}[F_n(y_n-1)]}^{\Phi^{-1}[F_n(y_n)]} (2\pi)^{-n/2}\exp\left(-\frac{1}{2}\boldsymbol{z}'\boldsymbol{z}\right) d\boldsymbol{z} = \prod_{i=1}^{n} P(Y_i = y_i).$$

Then (3) is

$$\begin{aligned}
&E\left[(2\pi)^{-n/2}|\Sigma|^{-1/2}\gamma\exp\left\{-\frac{1}{2}\boldsymbol{Z}^{*\prime}(\Sigma^{-1}-\boldsymbol{I})\boldsymbol{Z}^*\right\}\right] \\
&= (2\pi)^{-n/2}|\Sigma|^{-1/2}\gamma\int_{\Phi^{-1}[F_1(y_1-1)]}^{\Phi^{-1}[F_1(y_1)]} \cdots \int_{\Phi^{-1}[F_n(y_n-1)]}^{\Phi^{-1}[F_n(y_n)]} \exp\left\{-\frac{1}{2}\boldsymbol{z}'(\Sigma^{-1}-\boldsymbol{I})\boldsymbol{z}\right\}\gamma^{-1}\exp\left(-\frac{1}{2}\boldsymbol{z}'\boldsymbol{z}\right) d\boldsymbol{z} \\
&= (2\pi)^{-n/2}|\Sigma|^{-1/2}\int_{\Phi^{-1}[F_1(y_1-1)]}^{\Phi^{-1}[F_1(y_1)]} \cdots \int_{\Phi^{-1}[F_n(y_n-1)]}^{\Phi^{-1}[F_n(y_n)]} \exp\left(-\frac{1}{2}\boldsymbol{z}'\Sigma^{-1}\boldsymbol{z}\right) d\boldsymbol{z}
\end{aligned}$$

which is equal to (2).